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Anisotropic transport for the $\nu = 2/5$ fractional quantum Hall state at intermediate magnetic fields

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Abstract

The $\nu = 2/5$ state is spin unpolarized at weak magnetic field and fully polarized at strong field. At intermediate field, a plateau of half the maximal polarization is observed. We study this phenomenon in the framework of composite fermion (CF) theory. Due to the mixing of the CF Landau levels (LLs), the unidirectional charge/spin density wave state of CFs is lower in energy than the Wigner crystal. This means that transport anisotropy, similar to that for electrons in higher Landau levels at half-filling, may appear in this fractional quantum Hall state when the external magnetic field is in an appropriate range. When the magnetic field is tilted, the easy transport direction is perpendicular to the direction of the in-plane field. Varying the partial filling factor of CF LLs from 0 to 1, we find that the energy minimum occurs in the vicinity of a half.

The Coulomb interaction between electrons in two dimensions plays a dominant role when the kinetic energy is quenched into a series of degenerate Landau levels (LLs), and gives rise to a variety of unusual phenomena. The most famous example is the fractional quantum Hall (FQH) effect [1], where at certain filling factors the system condenses into the so-called Laughlin liquid which acquires gaps where charge excitations can appear. Another manifestation of Coulomb interaction is the formation of unidirectional charge density wave (UCDW) or stripe phases in high LLs at half-filling [2–5]. A unified description of all the fractional quantum states in the lowest LLs $\nu = n/(2pn + 1)$ was achieved in the composite fermion (CF) picture, in which each electron carries $2p$ statistical magnetic quanta, forming a composite particle [6, 7]. At a mean-field level, the CFs experience an effective magnetic field $B^* = B/(2pn + 1)$, in which they fill n CF LLs, and exhibit integer quantum Hall effects.

In recent years, the spin polarizations of electrons for integral as well as FQH states have been studied extensively. Many novel and interesting spin-related phenomena have been revealed. A large amount of data have been accumulated for various fractional filling states [8–14] as well as for $\nu = 1$ [15]. In a tilted magnetic field experiment carried out by Pan *et al* [16], the transport becomes highly anisotropic for even-number filling factors ($\nu = 4, 6, \ldots$)

when the tilted angle exceeds a large critical value. A new phase called the spin density wave (SDW) was proposed to explain this phenomenon [16, 17].

It is now well established that for some filling factors ($\nu = 1/m$, m an odd number) the ground state is fully spin polarized for all values of the Zeeman splitting, while for other filling factors (for example, $\nu = 2/5, 3/7$) the ground state is fully polarized only for large values of the Zeeman splitting but unpolarized for small (or zero) values of the Zeeman energy. One interesting observation is that at intermediate values of the Zeeman energy there appears a plateau of half the maximal spin polarization for $\nu = 2/5$ [18]. The stability of the half-polarized state implies that the ground state energy of the system as a function of spin polarization has nonmonotonic behaviour at half-polarization. The Zeeman energy $E_Z = g\mu_B B_{tot}$ favours spin polarization, while the electron–electron interaction favours a singlet state. However, the origin of the half-polarization phenomenon remains controversial. According to the theory of CFs with a spin, each CF LL is split into two subbands. As the $n = 0$ $\uparrow$-spin band is always fully occupied, it can be treated as a nondynamical background. At intermediate E_Z, the $n = 0$ $\downarrow$-spin CF LL and the $n = 1$ $\uparrow$-spin CF LL can both be partially occupied, with filling factors ν_1 and ν_2, respectively. $\nu_1 + \nu_2 = 1$. The half-polarized state corresponds to $\nu_1 = \nu_2 = 1/2$. The CFs occupying the partially filled CF LLs can be thought of as a system consisting of fermions of two types. Apalkov *et al* [19] mapped the two-component fermion system onto a system of excitons and described the ground state as a liquid state of excitons with nonzero values of the exciton angular momentum. Their calculation reveals that a downward cusp occurs at half the maximal spin polarization. On the other hand, Murthy [20] proposed a partially polarized density wave (PPDW) state of CFs on the basis of a Hartree–Fock (HF) theory developed earlier [22]. He compared the ground state energies of the PPDW state, in which one set of CDWs was placed directly over the other, with the Halperin (1, 1, 1) liquid state [23] and found that the PPDW state is slightly lower in energy than Halperin (1, 1, 1) state [20, 30].

Motivated by the above work, we compare the cohesive energies of the CDW and the UCDW states for CF LL filling factor $\nu = 2$. Allowing for different stacking possibilities for the two sets of interacting CDW or UCDW states consisting of type-1 and type-2 fermions, we find that one set of lattices is rigidly displaced with respect to the other. Since the two types of fermion have opposite spin, the shifted lattices form an antiferromagnet-like structure. This is called the SDW. We carry out a HF computation for such a system, with the external magnetic field tilted to a variety of angles. The results show that the cohesive energy of the shifted UCDWs is consistently lower than that of the shifted Wigner crystals (WCs), which means that the magnetotransport anisotropy, similar to those for electrons in high LLs at half-filling [2, 3], may appear in the lowest LL at $\nu = 2/5$ provided that the Zeeman energy is adjusted to an appropriate range. The easy transport direction is always perpendicular to the direction of the in-plane field. We also calculate the dependence of the cohesive energy on the partial filling factor ν_1. The result shows that the energy minimum appears at $\nu_1 = 1/2$, which implies that the half-and-half occupation of the $n = 0$ $\downarrow$-spin CF LL and the $n = 1$ $\uparrow$-spin CF LL is the most preferable state.

Suppose the two-dimensional electron system is confined by a harmonic potential in the z-direction with the characteristic frequency Ω. The external magnetic field is tilted to an angle θ with $\boldsymbol{B} = (B\tan\theta, 0, B)$. According to Murthy *et al* [21], the HF theory in terms of CF variables gives a reasonably good account of physical properties. The wavefunctions for the nth CF LL are

$$\phi_{n,X}(\boldsymbol{r}) = \frac{1}{\sqrt{L_y}} \mathrm{e}^{-\mathrm{i}Xy/l^2} \Phi_0^{\omega_+}((x - X)\sin\tilde{\theta} + z\cos\tilde{\theta})\Phi_n^{\omega_-}((x - X)\cos\tilde{\theta} - z\sin\tilde{\theta}), \tag{1}$$

where l is the magnetic length in the effective field and X is an integer multiple of $2\pi l^2/L$. Φ_n^ω is the harmonic oscillator wavefunction corresponding to the frequency ω and $\tan\tilde{\theta} = \frac{\omega_c^2}{\omega_+^2-\omega_c^2}\tan\theta$. The frequency $\omega_\pm$ are given by

$$\omega_\pm^2 = \frac{1}{2}\left(\Omega^2 + \frac{\omega_c^2}{\cos^2\theta}\right) \pm \sqrt{\frac{1}{4}\left(\Omega^2 - \frac{\omega_c^2}{\cos^2\theta}\right)^2 + \Omega^2\omega_c^2\tan^2\theta}. \tag{2}$$

The electron density operator is expressed in the momentum space as

$$\hat{\rho}(\boldsymbol{q}) = \sum_{n,n',X} \mathrm{e}^{-\mathrm{i}q_x X} c^\dagger_{n,X_-} c_{n',X_+}\rho_{nn'}(\boldsymbol{q}), \tag{3}$$

where $X_\pm = X \pm q_y l^2/2$ and $c_{n,X}$ destroys a CF in the state $\phi_{n,X}$. The matrix element $\rho_{nn'}(\boldsymbol{q})$ can be computed with the one-particle state equation (1). The Hamiltonian can now be written as [22]

$$H = \frac{1}{2L_xL_y}\sum_{\boldsymbol{q},\{n_i\},\{X_i\}} v(q)\mathrm{e}^{-\mathrm{i}q_x(X_1-X_2)}\rho_{n_1n_2}(\boldsymbol{q})\rho_{n_3n_4}(-\boldsymbol{q})c^\dagger_{n_1,X_{1-}}c_{n_2,X_{1+}}c^\dagger_{n_3,X_{2+}}c_{n_4,X_{2-}}, \tag{4}$$

where $v(q) = 2\pi e^2/q$. Equation (4) is the correct form for the CF Hamiltonian, and the energy coming from normal ordering represents the Hartree interaction of an electron with its own correlation hole.

By using the standard manipulation, we can explicitly write the effective potential $U_{nn'}(\boldsymbol{q})$ as a sum of a Hartree term (in units of $e^2/\kappa_0 l$):

$$H_{nn'}(\boldsymbol{q}) = \int \frac{\mathrm{d}q_z}{\pi l}\frac{1}{q_\parallel^2 + q_z^2}[F^\theta_{nn'}(\boldsymbol{q})]^2 \tag{5}$$

and a Fock term:

$$X_{nn'}(\boldsymbol{q}) = -2\pi l^2\int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^2}H_{nn'}(\boldsymbol{p})\mathrm{e}^{\mathrm{i}\boldsymbol{p}\times\boldsymbol{q}l^2}, \tag{6}$$

where $F^\theta_{nn'}(\boldsymbol{q})$ is given by

$$F^\theta_{nn'}(\boldsymbol{q}) = (n'!/n!)^{1/2}(\alpha^2/2)^{(n-n')/2}\mathrm{e}^{-\gamma^2/4-\alpha^2/4}L_{n'}^{n-n'}(\alpha^2/2) \tag{7}$$

with

$$\begin{aligned}\alpha^2 &= (q_x\cos\tilde{\theta} - q_z\sin\tilde{\theta})^2 l_-^2 + q_y^2 l^4/l_-^2\cos^2\tilde{\theta}\\ \gamma^2 &= (q_z\cos\tilde{\theta} + q_x\sin\tilde{\theta})^2 l_+^2 + q_y^2 l^4/l_+^2\sin^2\tilde{\theta}.\end{aligned} \tag{8}$$

Here $l_\pm^2 = \hbar/m\omega_\pm$ and $L_n^m(x)$ is the Laguerre polynomial.

We adopt the HF approximation in a similar form introduced by Côté and MacDonald for double-layer systems [25, 26], but the layer index is replaced by the type index in our case.

Allowing the charge density wave by making the ansatz

$$\langle c^\dagger_{nX-Q_yl^2/2}c_{n'X+Q_yl^2/2}\rangle = \mathrm{e}^{\mathrm{i}Q_xX}\Delta_{nn'}(\boldsymbol{Q}), \tag{9}$$

we carry out a HF computation on the UCDW and the triangular lattice. We assume that Δ_{nn} are nonzero only for $n = 0, 1$ [20]. The cohesive energy can be calculated in the same way as has been done for the WC [27–29]:

$$E_{coh} = \tfrac{1}{2}\sum_{Q\neq 0}\{U_{00}(\boldsymbol{Q}) + U_{11}(\boldsymbol{Q}) + 2\cos(\boldsymbol{Q}\cdot\boldsymbol{a})H_{01}(\boldsymbol{Q})\}|\Delta_{00}(\boldsymbol{Q})|^2, \tag{10}$$

where $\boldsymbol{a}$ is the relative shift of the two sets of lattices. Here we have not included the Zeeman energy and the uniform direct interactions. The latter are cancelled by the neutralizing positive

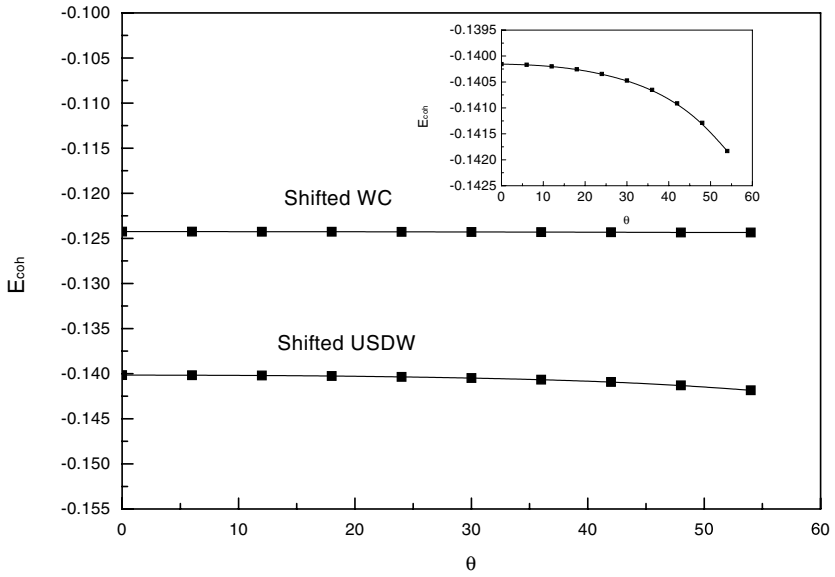

Figure 1. The cohesive energies (in units of $e^2/\kappa_0 l$) of shifted USDWs and shifted WCs versus the tilt angle θ for $\Omega/\omega_c = 3.0$. The inset has rescaled energy for the USDWs. It can be seen that the cohesive energy of the USDW state decreases as the tilt angle increases.

backgrounds. The interactions of the type-1 and type-2 fermions with the CFs in the fully filled lowest $n = 0$ $\uparrow$-spin CF LL subband are also omitted.

In our computations, the relative shift of the two sets of CDWs reduces the energy significantly. For the triangular lattice in the perpendicular magnetic field, the cohesive energy of the unshifted lattices is -0.0961 (in units of $e^2/\kappa_0 l$). The relative displacement ($\boldsymbol{a} = \{\frac{1}{2}\Lambda_b, \frac{\sqrt{3}}{6}\Lambda_b\}$ with lattice constant Λ_b) reduces the energy to -0.1242. For the UCDW state, the energy reduction for the shifted lattice is even larger. This reduction in energy for the shifted lattice is attributed mainly to the lack of exchange symmetry between the type-1 and type-2 fermions. Figure 1 shows that the shifted UCDW state is always preferable to the shifted WC state for all of the tilted angles. As in the case for high LLs, a stable UCDW ground state or stripes leads to the anisotropy in the magnetotransport experiment. As each set of the UCDWs has opposite spin polarization, the shifted lattice forms an antiferromagnetic pattern. We call this new phase the unidirectional spin density wave (USDW). The space period of the USDW is $\sim 5.2l$. For completeness, we have also considered the triangular lattice of 'bubbles', with each bubble containing in general several electrons, by using the scheme developed by Fogler *et al* [31]. Our result shows that the lattice with one electron per 'bubble' has the lowest cohesive energy.

In figure 2 we show our cohesive energy for the shifted UCDW calculated by varying the partial filling factor ν_1 from 0 to 1 for zero in-plane magnetic field. We assume that the space periods of the CDWs are the same for both CF LL bands. It can be seen that the energy minimum appears at half-filling, which means $\nu_1 = \nu_2$. This figure reflects the typical characteristics of figure 1 in [19]. It indicates that the state of half-and-half occupation for the $n = 0$ $\downarrow$-spin CF LL and the $n = 1$ $\uparrow$-spin CF LL is energetically more stable than the full occupation of just one subband of the CF LLs. Taking account of the fully occupied $n = 0$ $\uparrow$-spin band, the total polarization is a half.

Because of the small g-factor of electrons in GaAs, spins may not be fully polarized in FQH states. Transitions between singlet, partially polarized, and fully polarized states for a

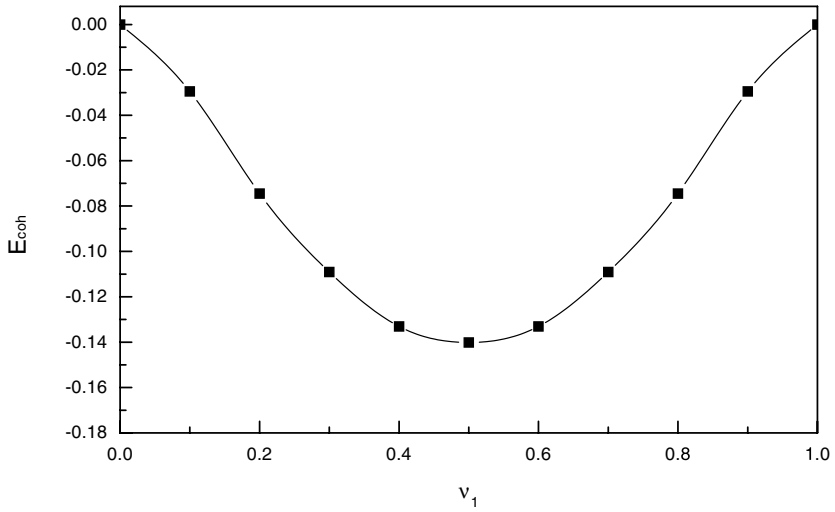

Figure 2. The cohesive energy E_{coh} of the USDW versus the partial filling factor ν_1. The minimum of energy at $\nu_1 = 1/2$ indicates that the half-and-half occupation of CF spin subbands is energetically preferable to the full occupation of just one CF spin subband.

number of fractional fillings can be understood in terms of CFs with a spin [6, 9, 12]. For $\nu = 2/5$, the transition takes place when the unoccupied $n = 1$ $\uparrow$-spin CF LL subband crosses the occupied $n = 0$ $\downarrow$-spin CF LL subband as the Zeeman energy increases. If the crossing is trivial, nothing interesting will happen. However, the competition between exchange and direct Coulomb interaction for CFs results in spontaneous breakdown of the translational symmetry.

The mechanism for the USDW in this work is to some extent similar to that for the isospin stripes at integer filling factors for the double-layer systems at $\nu = 4n + 1$ [33], in which the real spin index is replaced by the isospin index. The origin of such isospin stripe order is a competition between the exchange and the direct Coulomb interaction. Exchange favours accumulating all the electrons in one layer (in order to maximize the isospin exchange field), whereas direct Coulomb energy is lower when the electron density is distributed uniformly between the layers. It is also interesting to compare our prediction of the USDW to the anisotropic transport for even-number filling factors ($\nu = 4, 6, \ldots$) when the magnetic field is tilted to a large angle [16]. It can easily be seen that the USDW in this work is just a CF version of the USDW of electrons suggested in [16, 17].

Hitherto, transport anisotropy was experimentally observed only in high LLs [2, 3]. Our results predict that in the lowest LL, there may also exist stripe phases at some fractional filling factors ($\nu = 2/5, 3/7, \ldots$), provided that the Zeeman energy E_Z is set to an appropriate value. Since the space period of the USDW is large enough, the USDW is stable against the quantum fluctuations in the lowest LL. We note that Lee *et al* [32] proposed another kind of spontaneous stripe order at certain even-denominator fractions in the lowest LL. They argued that for LL filling factors of the form $\nu = (2n + 1)/(4n + 4)$, which correspond to CF filling factors $\nu^* = n + 1/2$, the system phase separates into stripes of n and $n + 1$ filled CF LLs. Our picture of the USDW is fundamentally different to theirs.

In summary, we have computed the ground state energies of USDWs and WCs consisting of CFs. We find that the relative shift of the two sets of interacting CDW lattices remarkably reduces the cohesive energy of the PPDW state of CFs proposed by Murthy [21]. When the external magnetic field is tilted, the USDW is always energetically preferable, which means that anisotropic transport may be observed in the lowest LL at $\nu = 2/5$ if the Zeeman energy

is properly adjusted. We also found that half-and-half occupation of the $n = 0$ $\downarrow$-spin CF LLs and the $n = 1$ $\uparrow$-spin CF LLs is more stable than the full one-band occupation. Experimental observation of stripes in the lowest LL would support the concept of such antiferromagnet-like charge/spin density waves for CFs that we have described in this work.

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